

$$a) \quad f(x_1, x_2) = \begin{pmatrix} 5x_1 x_2 \\ x_1^2 x_2^2 + x_1 + 2x_2 \end{pmatrix}, \quad \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$f_1 = 5x_1 x_2$$

$$\frac{\partial f_1}{\partial x_1} = 5x_2$$

$$\frac{\partial f_1}{\partial x_2} = 5x_1$$

$$f_2 = x_1^2 x_2^2 + x_1 + 2x_2$$

$$\frac{\partial f_2}{\partial x_1} = 2x_1 x_2^2 + 1$$

$$\frac{\partial f_2}{\partial x_2} = 2x_1^2 x_2 + 2$$

$$x_1 = 1, \quad x_2 = 2$$

$$\frac{\partial f_1}{\partial x_1} = 5(2) = 10$$

$$\frac{\partial f_1}{\partial x_2} = 5(1) = 5$$

$$\frac{\partial f_2}{\partial x_1} = 2(1)(2^2) + 1 = 9$$

$$\frac{\partial f_2}{\partial x_2} = 2(1^2)(2) + 2 = 4 + 2 = 6$$

$$Df(1, 2) = \begin{pmatrix} 10 & 5 \\ 9 & 6 \end{pmatrix}$$

$$b) \quad f(x_1, x_2, x_3) = \begin{pmatrix} \ln(x_1^2 + x_2^2) + x_3^2 \\ \exp(x_1^2 + x_3^2) + x_1 \\ \frac{1}{x_2^2 + x_1^2} + x_2^2 \end{pmatrix}, \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$f_1 = \ln(x_1^2 + x_2^2) + x_3^2$$

$$\frac{\partial f_1}{\partial x_1} = \frac{2x_1}{x_1^2 + x_2^2} \Rightarrow \frac{2(1)}{1+4} = \frac{2}{5} = 0,4$$

$$\frac{\partial f_1}{\partial x_2} = \frac{2x_2}{x_1^2 + x_2^2} \Rightarrow \frac{2(2)}{1+4} = \frac{4}{5} = 0,8$$

$$\frac{\partial f_1}{\partial x_3} = 2x_3 \Rightarrow 2(3) = 6$$

$$f_1 = \exp(x_2^2 + x_3^2 + x_1^2)$$

$$\frac{\partial f_1}{\partial x_1} = 2x_1 \Rightarrow 2(1) = 2$$

$$\frac{\partial f_1}{\partial x_2} = 2x_2 e^{x_2^2 + x_3^2} \Rightarrow 2(2) e^{4+9} = 4e^{13}$$

$$\frac{\partial f_1}{\partial x_3} = 2x_3 e^{x_2^2 + x_3^2} \Rightarrow 2(3) e^{4+9} = 6e^{13}$$

$$f_3 = (x_3^2 + x_1^2)^{-1} + x_2^2$$

$$\frac{\partial f_3}{\partial x_1} = -1(x_3^2 + x_1^2)^{-2} \cdot 2x_1 = \frac{-2x_1}{(x_3^2 + x_1^2)^2} \Rightarrow \frac{-2}{100} = -0,02$$

$$\frac{\partial f_3}{\partial x_2} = 2x_2 \Rightarrow 2(2) = 4$$

$$\frac{\partial f_3}{\partial x_3} = \frac{-2x_3}{(x_3^2 + x_1^2)^2} \Rightarrow \frac{-6}{100} = -0,06$$

$$Df(1, 2, 3) = \begin{pmatrix} 0,4 & 0,8 & 6 \\ 2 & 4e^{13} & 6e^{13} \\ -0,02 & 4 & -0,06 \end{pmatrix}$$