

x_i	4	6	8	10
y_i	4	7	9	0

$$h_0 = h_1 = h_2 = 2$$

$$S_i(x) = a_i + h_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3 \quad i=0,1,2$$

$$M_i = S''(x_i)$$

1)

$$M_0 = 0 \quad M_3 = 0$$

$$h_{i-1}M_{i-1} + 2(h_{i-1} + h_i)M_i + h_i M_{i+1} = 6 \left(\frac{y_{i+1} - y_i}{h_i} - \frac{y_i - y_{i-1}}{h_{i-1}} \right)$$

$i=1$

$$2M_1 + 8M_2 + 2M_3 = 6 \left(\frac{9-7}{2} - \frac{7-6}{2} \right)$$

$$2M_1 + 2M_2 = 27$$

$i=2$

$$2M_1 + 8M_2 + 2M_3 = 6 \left(\frac{0-9}{2} - \frac{9-7}{2} \right)$$

$$2M_1 + 8M_2 = -45$$

$$\begin{cases} 8M_1 + 2M_2 = 27 \\ 2M_1 + 8M_2 = -45 \end{cases} \Rightarrow \begin{cases} 4M_1 + M_2 = \frac{27}{2} \\ M_1 + 4M_2 = -\frac{45}{2} \end{cases}$$

$$M_1 = \frac{51}{10} \quad M_2 = -\frac{69}{10}$$

$$M_0 = 0, \quad M_1 = \frac{51}{10}, \quad M_2 = -\frac{69}{10}, \quad M_3 = 0$$

2)

$$a_i = y_i, \quad c_i = \frac{M_i}{2}, \quad d_i = \frac{M_{i+1} - M_i}{6h_i}$$

$$h_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{6} (2M_i + M_{i+1})$$

$$i=0 \quad [4; 6]$$

$$a_0 = 6$$

$$h_0 = \frac{3-6}{2} - \frac{2}{6} \left(0 + \frac{51}{10} \right) = -\frac{3}{2} - \frac{17}{10} = -\frac{16}{5}$$

$$c_0 = 0 \quad d_0 = \frac{\frac{51}{10} - 0}{12} = \frac{17}{40}$$

$$S_0(x) = 6 - \frac{16}{5}(x-4) + \frac{17}{40}(x-4)^3$$

$$i=1 \quad [6; 8]$$

$$a_1 = 3$$

$$h_1 = \frac{9-3}{2} - \frac{2}{6} \left(2 \cdot \frac{51}{10} - \frac{69}{10} \right) = 3 - \frac{17}{10} = \frac{19}{10}$$

$$c_1 = \frac{51}{20} \quad d_1 = \frac{-\frac{69}{10} - \frac{51}{10}}{12} = -1$$

$$S_1(x) = 3 + \frac{19}{10}(x-6) + \frac{51}{20}(x-6)^2 - (x-6)^3$$

$$i=2 \quad [8; 10]$$

$$a_2 = 9$$

$$h_2 = \frac{0-9}{2} - \frac{2}{6} \left(2 \cdot \left(-\frac{69}{10} \right) + 0 \right) = -\frac{9}{2} + \frac{66}{10} = \frac{1}{10}$$

$$c_2 = -\frac{69}{20} \quad d_2 = \frac{0 - \left(-\frac{69}{10} \right)}{12} = \frac{23}{40}$$

$$S_2(x) = 9 + \frac{1}{10}(x-8) - \frac{69}{20}(x-8)^2 + \frac{23}{40}(x-8)^3$$

$$S(x) = \begin{cases} 6 - \frac{16}{5}(x-4) + \frac{17}{40}(x-4)^3 & 4 \leq x \leq 6 \\ 3 + \frac{19}{10}(x-6) + \frac{51}{20}(x-6)^2 - (x-6)^3 & 6 \leq x \leq 8 \\ 9 + \frac{1}{10}(x-8) - \frac{69}{20}(x-8)^2 + \frac{23}{40}(x-8)^3 & 8 \leq x \leq 10 \end{cases}$$

$$a_0 = 6$$

$$h_0 = -\frac{16}{5}$$

$$c_0 = 0$$

$$d_0 = \frac{17}{40}$$

$$a_1 = 3$$

$$h_1 = \frac{19}{10}$$

$$c_1 = \frac{51}{20}$$

$$d_1 = -1$$

$$a_2 = 9$$

$$h_2 = \frac{1}{10}$$

$$c_2 = -\frac{69}{20}$$

$$d_2 = \frac{23}{40}$$

